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FIXED POINT THEOREM IN FUZZY METRIC SPACE

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ABSTRACT
In this paper,we prove a fixed point theorem using compatible condition . Our result generalizes the result of Kang and Rhoades satisfying contractive condition of integral type.

Keywords- *common fixed point ,compatible mappings of type(P).*

I. INTRODUCTION

Rhoades(1985)proved for pair of mappings which in turn was generalized by Kang and Rhoades (1996)using compatible condition defined by Jungck(1986).Branciari[1] obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an intregral-type inequality.The second auther [6] prove Two fixed point theorems involving more general contractive conditions.

II. PRELIMINARIES

Consider $\Phi = \{ \varphi : \varphi : \Re_+ \to \Re \}$ such that φ is nonnegative,Lebesgue integrable,and satisfies $\int_0^{\varepsilon} \varphi(t)dt > 0$ for each $\varepsilon > 0$ (2.1)

Let $\psi : \Re_+ \to \Re_+$ satisfy that

(i) ψ is nonnegative and nondecreasing on $\Re_+$,

(ii) $\psi(t)<t$ for each $t>0$,

(iii) $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each fixed $t>0$. where $\Psi = \{\psi : \psi$ satisfies (i)-(iii)$\}$.

III. MATERIAL AND METHODS

Theorem (3.1): Let A,B,S and T be the mappings from a complete metric space (X,d) into itself satisfying the following conditions:

S and T are surjective

(ii) One of A,B,S and T is continuous

(iii) A,S and B,T are compatible pairs of type(P)

(iv) $$\int_0^{M(Ax,By,z)} \varphi(t)dt \ge \psi\left(\int_0^{M(x,y,z)} \varphi(t)dt\right) \qquad (3.1)$$

Wherek $\in [0,1), \varphi \in \Phi, \psi \in \Psi$,and

M(x,y,z)=max{M(Sx,Ty,z),M(Sx,Ax,z),M(Ty,By,z),

$$\frac{\max\{M(Sx,Ty,z).M(Sx,By,z),M(Sx,Ty,z).M(Ty,Ax,z),M(Sx,Ax,z).M(Sx,By,z),M(Ty,By,z).M(Ty,Ax,z)\}}{\max\{d(Sx,By,z),M(Ty,Ax,z)\}} \qquad (3.2)$$

when M(x,y,z)=1 if Sx=By and Ty=Ax.Then A,B,S and T have a unique common fixed point in X.

IV. RESULT AND DISCUSSION

Proof of theorem:If A,B,S and T be the mappings from a complete metric space (X,d)into itself then there exists a sequence $\{x_n\} \subset X$ with $x_0 \in X$,

$Ax_{2n} = Tx_{2n+1} = x_{2n+1}$ and $Bx_{2n+1} = Sx_{2n+2} = x_{2n+2}$

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Now assume x2n+ $\neq$ x2n+2 for each n.With x=x2n, y= x2n+1,then from(3.1) we have

$$\int_0^{M(Ax_{2n},Bx_{2n+1},z)} \varphi(t)dt \geq \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt\right.$$

$$\Rightarrow \int_0^{M(x_{2n+1},x_{2n+2},z)} \varphi(t)dt \geq \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt\right. \qquad (3.3)$$

Continuing this process,we have

$$\int_0^{M(Ax_{2n},Bx_{2n+1},z)} \varphi(t)dt \geq \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt \geq \ldots \leq \psi\right. 2n(d) \qquad (3.4)$$

where d= $\int_0^{M(x_0,x_1,z)} \varphi(t)dt$. Then it is easily shown that {xn} is Cauchy,hence convergent.Call the limit p.Consequently the subsequences{Ax2n},{Bx2n+1}, {Sx2n}, {Tx2n+1} converge to p .

Let p=Sp=Ap .Then M(p,p,z)

= max {M(Sp,Tp,z),M(Sp,Ap,z),M(Tp,Bp,z),

$$\frac{\max\{M(Sp,Tp,z).M(Sp,Bp,z),M(Sp,Tp,z).M(Tp,Ap,z),M(Sp,Ap,z).M(Sp,Bp,z),M(Tp,Bp,z).M(Tp,Ap,z)}{\max\{M(Sp,Bp,z),M(Tp,Ap,z)\}}\} \qquad (3.5)$$

Therefore M(p,p,z)= max {M(p,Tp,z),1,M(Tp,Bp,z),

$$\frac{\max\{M(p,Tp,z).M(p,Bp,z),M(p,Tp,z).M(Tp,p,z),1.M(p,Bp,z),M(Tp,Bp,z).M(Tp,p,z)\}}{\max\{M(p,Bp,z),M(Tp,p,z)\}} \qquad (3.6)$$

Hence M(p,Tp,z).M(p,Bp,z)=1 since M(Tp,Bp,z) ≥M(Tp,p,z)*M(p,Bp,z)

So p=Tp ,p=Bp and (3.1) becomes

$$\int_0^{M(p,Tp,z)} \varphi(t)dt \geq \psi\left(\int_0^{M(p,Tp,z)} \varphi(t)dt\right. \qquad (3.7)$$

which, from (3.3) ,implies that p=Tp=Bp.

Similarly , p=Tp=Bp implies that p=Sp=Ap. We will now show that A,B,S and T satisfy (3.5). M(x,Sx,z)=max M(Sx,TSx,z),M(Sx,Ax,z),M(TSx,BSx,z),

$$\frac{\max\{M(Sx,TSx,z)M(Sx,BSx,z),M(Sx,TSx,z)M(TSx,Ax,z),M(Sx,Ax,z)M(Sx,BSx,z),M(TSx,BSx,z)M(TSx,Ax,z)\}}{\max\{M(Sx,BSx,z),M(TSx,Ax,z)\}} \qquad (3.8).$$

Now we have M(Sx,TSx,z) ≥M(Sx,Ax,z)*M(Ax,TSx,z)

M(Sx,BSx,z)≥ M(Sx,Ax,z)*M(Ax,BSx,z)

M(BSx,TSx,z) ≥M(BSx,Bx,z)*M(Bx,TSx,z)

on the considering above we have

$$\int_0^{M(Sx,TSx,z)} \varphi(t)dt \geq \psi\left(\int_0^{M(Sx,TSx,z)} \varphi(t)dt\right. \quad \text{a contradiction}$$

V. CONCLUSION

Hence S,A and T,B has a fixed point ,that any fixed point of S,A is also a fixed point of T,B and conversely.Thus S,A and T,B have a common fixed point . Hence A,B,S and T has a common fixed point .

Suppose that T is continous. Since B and T are compatible,then

TTx2n+1 =Tp and BBx2n+1=Tp .

So that M(TTx2n+1 ,BBx2n+1,z)=1=M(SSx2n,AAx2n,z) as n $\rightarrow \infty$

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